A Mathematician's Apology Study Guide

A Mathematician's Apology by G. H. Hardy

BOOKRAGS

Contents

Introduction

Godfrey Harold (G. H.) Hardy's *A Mathematician's Apology*, first published in 1940 in England, is the memoir of the world-renowned mathematician, written in the last few years of his life while he was in failing health. The work is written in the form of an apology, which in literary terms means a defense. In this case, Hardy is defending his career as a theoretical mathematician. To make the defense comprehensible to the layperson, Hardy discards the language he would use in an academic paper and instead adopts a succinct and simple writing style aimed at a general audience. The book is not mathematical; rather, it is an affirmation of a career that happens to be mathematical and purely speculative.

It should be noted that Hardy speaks exclusively of men in his writing, which reflects the secondary role women of his era played in the British university system in general and in the field of mathematics in particular. Hardy does not mention or refer to a single woman intellectual or a work by a woman

A Mathematician's Apology is a lasting testament to Hardy's passion for intellectual pursuits. Hardy likens mathematics to art and explains math in much the same way a critic explains art. He elaborates on the qualities of mathematical genius and the logical reasons for pursuing a career in mathematics, and he briefly outlines three of the most basic and timeless theorems in order to illustrate the inherent beauty of mathematics for the layperson. Many of the chapters also address the differences between theoretical or "pure" mathematics to which Hardy dedicated his life and several types of "applied math," which he regards as largely inferior. The work also reveals the grave doubts Hardy harbored about the overall usefulness of his work and life. While *A Mathematician's Apology* has had an enormous influence on generations of mathematicians, it has also been viewed by many as a psychological document of a genius with depressive tendencies. As Hardy contemporary C. P. Snow acknowledges in the book's introduction, *A Mathematician's Apology* "is a book of haunting sadness."

Author Biography

Godfrey Harold (G. H.) Hardy was born on February 7, 1877, in Cranleigh, Surrey, England. Both his parents were educators and possessed mathematical skills. Even before learning to speak as a very young child, he demonstrated an extraordinary IQ and performed mathematical computations to amuse himself. After winning a scholarship to Winchester College in 1889, Hardy began the rigorous training of a mathematician.

In 1896, he entered Trinity College, Cambridge, where he trained under A. E. H. Love, who gave him his first serious conception of analysis by introducing him to Camille Jordan's *Cours d'analyse*. Thereafter, Hardy committed his life to mathematics, and by 1908 he had already made a significant contribution, with his greatest work in this early period being *A Course of Pure Mathematics*.

A watershed year for Hardy was 1911, as it marked the beginning of his thirty-five-year collaboration with fellow mathematician J. E. Littlewood. Two years later, in 1913, he received an unsolicited manuscript from Indian mathematician Srinivasa Ramanujan. Hardy immediately spotted Ramanujan's genius and brought him to Cambridge where, between 1914 and 1918, the men engaged in what would become one of mathematics' most remarkable collaborations.

It was during the years of World War I that Hardy also became known for his outspoken political views. Unlike most of his contemporaries and colleagues, Hardy held the Germans in high regard for their intellectual prowess and contributions to scientific thought. His ingrained distrust of British politicians contributed to his deep anger at Great Britain's participation in the war. He was particularly upset over the interruption it caused in his various collaborations with colleagues outside England.

In 1919, Hardy left Cambridge for a position as the Savilian professor of geometry at Oxford, where he remained until 1931, at which time he returned to Cambridge, where he finished his professional career. An avid cricket fan and tennis player, Hardy remained physically active throughout his life until 1939 when, at the age of sixty-two, he had a heart attack. His remarkable mental powers quickly began to leave him and sports became impossible. He was also filled with anger that Europe had again entered into war. However, Hardy had one further gift to leave to the world, namely *A Mathematician's Apology*, published in 1940, which has inspired many people towards mathematics.

By the time World War II ended in 1945, Hardy's health was failing fast, as was his creativity. He gradually became depressed, and in early summer 1947, he unsuccessfully tried to take his own life by taking a large dose of barbiturates. He took so many, however, that he became sick before he died, and he was resuscitated and survived.

Hardy, who became almost as well known for his outspoken beliefs and rebellious spirit as for his mathematical skills, once listed among his most ardent wishes: 1) To prove the Riemann hypothesis (a famous unsolved mathematical problem); 2) to make a brilliant play in a crucial cricket match; 3) to prove the nonexistence of God; and 4) to murder Mussolini, the Italian fascist leader (Hoffman, *The Man Who Only Knew Numbers*).

Over the course of his lifetime, Hardy received many honors for his work. He was elected a fellow of the Royal Society in 1910, and he received the Royal Medal of the society in 1920 and the Sylvester Medal of the society in 1940. On December 1, 1947, shortly after hearing that he was to be given the Copley Medal, the highest honor of the Royal Society, Hardy passed away in Cambridge, Cambridgeshire, England.

Plot Summary

Chapters 1-2

Hardy opens his apology by asserting his belief that in the mere act of "writing about mathematics" he has lowered himself to a level below that of a pure mathematician. He equates himself in this position to that of an art critic—a profession he considers to be for "second-rate minds"—as opposed to the artist himself. Hardy describes a discussion he had on this subject with British poet A. E. Houseman. In chapter 2, Hardy introduces the questions he proposes to answer throughout the remainder of the book: Why is it worthwhile to make a career out of mathematics? And what is the proper justification of a mathematician's life?

Chapters 3-4

Hardy states that most people choose their career path because "it is the one and only thing that [they] can do at all well." Mathematics is a particularly specialized subject, and mathematicians themselves are not noted for their versatility. In chapter 4, he lists several mathematicians whom he considers immortal geniuses, and he points out that most of them reached their intellectual peaks or died before the age of forty. Those men who attempted new careers later in life were largely failures. Hardy uses these points to illustrate why he is now writing this memoir: simply put, he is too old to continue with theory, and he has no talent for any other career.

Chapters 5-9

Hardy concludes his responses to the questions he posed in chapter 3. As to why one would choose to become a mathematician, Hardy refers to a lecture he gave at Oxford twenty years earlier in which he posited that mathematics is chosen for three reasons. First, it is essentially a "harmless" profession; second, because the universe is so vast, if a few professors wasted their lives doing something at which they excelled, it would be "no overwhelming catastrophe"; and third, there is a "permanence" of mathematics that is "beyond the powers of the vast majority of men." It is here that Hardy adds what he believes are the three prime motivations that impel men to choose their professions: intellectual curiosity, professional pride, and ambition for reputation and the rewards it brings. To support these statements, Hardy lists several ancient civilizations that are long forgotten save for their mathematical discoveries. He concludes with a dream that mathematician and philosopher Bertrand Russell once related that expressed Russell's deep-seated fear that he would one day be forgotten by future generations.

Chapters 10-11

Hardy posits that mathematics has an aesthetic quality like that of art or poetry—a position for which he and this book are best remembered. Hardy takes a swipe at one of his contemporaries, mathematician Lancelot Hogben, who was well-known for his opposition to Hardy's theories. Hardy uses the example of chess to refute Hogben. Because chess is revered by the masses and is an exercise in pure mathematics, though admittedly of a "lowly kind," when one appreciates the beauty of a particular chess move, one is in essence appreciating its mathematical beauty. However, since the best mathematics also demands "seriousness," or "importance," and since no chess player or problem "has ever affected the general development of scientific thought," chess is "trivial" compared to pure mathematics.

Chapters 12-14

Hardy uses the examples of proofs by Euclid and Pythagoras to illustrate the beauty of mathematics and then explains why they are significant in spite of the fact that they are not practical. These proofs are presented concisely and demand only a rudimentary background in mathematics to follow them. It is the only instance in the memoir in which Hardy attempts to explain mathematical concepts or logic.

Chapters 15-18

Hardy continues to refine his concept of mathematical beauty by further defining the idea of "seriousness." To do this, he introduces the concepts of "generality" and "depth." Generality can be loosely defined as "abstractness," while depth is comparable to "difficulty." He discusses mathematician and philosopher Alfred North Whitehead's assertions, which he quotes: "The certainty of mathematics rests entirely on abstract generality." Hardy partially accepts this argument from a logician's point of view but argues that theoreticians must look for the difference among generalities. With regard to "depth," Hardy employs a metaphor that equates theorems with geologic strata: the more difficult the theorem, the more layers it has, with each layer holding ideas that link to other ideas above and below it. He concludes this discussion in chapter 18 by explaining why chess can never be "beautiful." In short, the very nature of chess demands that any given move can be answered with multiple countermoves—what Hardy refers to as "proof by enumeration of cases," which is the antithesis of beauty in a mathematician's eye. Rather than being a beautiful collection of mathematical theorems competing with one another, a chess game is, at its heart, a psychological battle between two intelligent beings.

Chapters 19-21

Hardy returns to his Oxford lecture in order to address the question of the usefulness of mathematics. In short, Hardy states emphatically that although some "elementary"

mathematics such as calculus have some utility, the "pure" mathematics with which he concerns himself cannot be justified on utilitarian grounds.

Chapters 22-24

Hardy returns to his comparison of "applied" and "theoretical" mathematics and states that it is a gross oversimplification to say that one has utility while the other does not. Hardy supports this statement by setting out to argue that pure mathematics is closer to "reality" than is applied mathematics. His assumption here is that there is a "mathematical reality" that exists that is no different from the "physical reality" to which most of us can relate. Mathematical reality is not a mental construct but rather an objective reality that exists in the world that can be discovered and described. Mathematicians who "create" proofs are actually doing little more than taking notes on their observations.

To illustrate this point, Hardy draws on the field of geology, which sets out to draw a "picture" of a part of mathematical reality. However, because geometry does not account for changes in spatiotemporal reality, such as those created by eclipses and earthquakes (since these are not mathematical concepts), the "drawing" a geometer creates in his theorems may suddenly have little to do with the physical reality surrounding him. However, the truths of the theorem remain unaffected. Or, to put it in even simpler terms, while spilling coffee on the pages of a Shakespeare play may make certain pages unreadable, the spill does not affect the play itself. By analogy, pure mathematicians concern themselves with the play, while applied mathematicians concern themselves with the pages on which the play is written.

In chapter 24, Hardy makes the seemingly paradoxical claim that despite these relationships, pure mathematicians are in fact the closer of the two to reality. Hardy's argument is as follows: an applied mathematician must work with a physical reality over which there is ample disagreement as to what comprises it. There is confusion as to what constitutes a chair, for instance: it may be a mass of whirling electrons, or it may be an "idea of God." The pure mathematician, however, works with a mathematical reality about which there is no ambiguity. No one disagrees as to what "2" or "317" is, and "317" is a prime number not because we "think it so" but rather because "it is so."

Chapter 25

Continuing the comparison of pure and applied mathematics, Hardy claims that pure mathematics is timeless, has a permanent aesthetic value, and its eternal qualities bring about a lasting sense of emotional satisfaction. The achievements of applied mathematicians, on the other hand, are more modern and temporal. Hardy leans towards calling the applied mathematical theories "useless."

Chapter 26

Hardy continues to delve into the idea of utility in mathematics, asking, "What part of mathematics are useful?" He goes on to list branches according to utility. He comes to the general conclusion that the more useful a type of mathematics is to an engineer or physicist, the less aesthetic value it has. Hardy prefers the world of imagination and art to the "humdrum" reality of applied mathematics. Hardy writes

'Imaginary' universes are so much more beautiful than this stupidly constructed 'real' one; and most of the finest productions of an applied mathematician's fancy must be rejected, as soon as they have been created, for the brutal but sufficient reason that they do not fit the facts.

Chapter 27-28

Hardy addresses some of the objections of his critics, especially applied mathematician Lancelot Hogben. Hardy's tone is snide and superior as he sums up his arguments regarding the differentiation between real and applied mathematics. He repeatedly uses the word "trivial" in reference to applied mathematics. According to Hardy, real mathematicians are artists. Hardy does not offer any justification of applied mathematics, saying only that it would appeal to Hogben. It is here that Hardy finally broaches the subject of utility and harm. Writing under the threat of an impending world war, Hardy feels that it is necessary to lead his discussion towards the relationship between mathematics and war. He comes to the conclusion that "real mathematics has no effects on war. No one has yet discovered any warlike purpose to be served by the theory of numbers or relativity, and it seems unlikely that anyone will do so for many years."Hardy wrote these words only five years before the theories of relativity helped the United States develop the first atomic bomb.

Chapter 29

Hardy concludes his memoir by returning to a more personal narrative voice. This final chapter is far more autobiographical than the rest of the memoir. Having already stated his theories, Hardy feels justified in summing up his life. It is the summation of a man knowingly in his declining years. Although the tone is sad and melancholic, he seems to convince himself that his life has had meaning. In lines often quoted by critics of the work, Hardy writes, "Well, I have done one thing *you* could never have done, and that is to have collaborated with both [mathematicians John Edensor] Littlewood and [Srinivasa] Ramanujan on something like equal terms." One quotes these lines so often because while so much of the work paints Hardy as pompous, this quotation is a clear illustration of his humility as well. He is able to recognize genius and also admit to his own limitations. In a work that carries a subtle sadness throughout it, these line spring forth as a positive affirmation of a genius' existence.

Detailed Summary & Analysis

Summary

A Mathematician's Apology is a memoir, the personal account of the life of Godfrey Harold (G.H.) Hardy, who, in failing health, looks back upon a life spent in the service of mathematics. Though an eminent mathematician, Hardy, in his sixties at the time of this book's writing, and of the mind that he was no longer able to make a significant contribution to the world of mathematics, attempts, on one level, to justify his existence, to explain to the layman, and perhaps even to himself, what it is that inspired him so.

In his attempt to justify the choice of a mathematician's life, Hardy initially poses two questions: whether the work a man does is worth doing, and why is it, exactly, that he does that work? To both he answers, as he believes most honest men would, "I do what I do because it is the one and only thing that I can do at all well." Hardy then continues by saying that sacrificing one's talent, regardless of what that may be, in pursuit of a vocation deemed more valuable is foolhardy and can be justified only by economic necessity or age. One hypothetical example he uses is that of Donald Bradman, regarded as the best batsman ever to have played the game of cricket, giving up his career to write poor poetry.

Mathematics, Hardy goes on to say, is, moreso than any other pursuit, a young man's game. Hardy cites numerous examples to that effect, many of which include the world's greatest mathematicians (Newton, et al) who performed their best work prior to the age of fifty.

Shortly thereafter, Hardy addresses the second question posed earlier, namely, 'why?' This, Hardy admits, is a far more difficult question to answer. Hardy remembers a lecture he gave at Oxford in 1920 and reiterates the pertinent points here. Hardy stresses the *harmlessness* of mathematics and emphasizes the permanence of mathematical achievement, but he also admits to being unable to answer the above mentioned questions at that moment, and says that he will have to come back to them later.

Ambition, Hardy goes on to stress, is a noble passion and the driving force behind much of the best work to be seen in the world. A mathematician, he expounds, has the best chances of satisfying the three criteria he regards as the incentives to research: intellectual curiosity, professional pride, and ambition. Hardy believes mathematical achievement is the most enduring of all achievements. History, he remarks, has a tendency to be unjust to its greater figures, though mathematicians, he concludes, are almost always remembered solely for their merit and not for any deplorable faculty.

Like both painters and poets, mathematicians, Hardy believes, create patterns, but with one clear distinction: a mathematician's patterns are based upon ideas. So too, he says,

though rather more intangible than those of the painter and poet, a mathematician's patterns must be beautiful, for there is no place for ugly mathematics.

From there, Hardy distinguishes between what he regards as 'trivial' mathematics and 'serious' mathematics; chess is used as an example of the former. According to Hardy, serious mathematics, or at least, the extent of seriousness, is bound inextricably to the significance, not of the theorem itself, but of the mathematical ideas which it connects.

Hardy presents two famous theorems of Greek mathematics, simple in both idea and execution but theorems of the highest quality nonetheless: Euclid's proof of the existence of an infinity of prime numbers and Pythagoras's proof of the irrationality of the square-root of two. Both of these theorems are proven, using the technique known as *reductio ad absurdum*. Hardy also mentions Fermat's 'two square' theorem as an example of 'beautiful' mathematics. Beauty and seriousness are the criteria upon which a mathematician's patterns should be judged, says Hardy, but he grapples ceaselessly with how to explain this notion of 'seriousness'

A serious theorem, he continues, is one in which the ideas are significant, though quantifying these ideas to the layperson is something he admits is rather difficult. However, he believes two things, *generality* and *depth,* are essential for a mathematical idea to be significant. The first criterion, Hardy muses, is corollary to abstractness and that it is, in essence, not the generality that is common to all theorems that is important, but rather, the differences in generality between them that is important. That is to say, while it is important that all theorems possess a generality and common nature, it is their differences, their uniqueness, that ultimately defines them.

Therefore, Hardy struggles with the definition of *depth*. While it has something to do with difficulty, it is not solely defined by the complexity of a problem. The ideas underlying Pythagoras's Theorem are deep, but Hardy supposes no mathematician would have any trouble proving them. Conversely, other theorems can be superficial and difficult to prove.

Though Hardy is primarily interested with mathematics as a creative art, he ponders the utility of mathematics and believes that science or art can be said to be useful if its development enhances the lives of mankind. Hardy concedes that math has its applications in engineering and physiology disciplines, but he finds it astonishing how little practical value scientific knowledge has for ordinary men. Hardy attempts to explain that elementary mathematics has considerable practical usefulness, but that 'pure' or 'real' mathematics, of which he is a proponent, is almost entirely useless, and the life of any 'pure' mathematician cannot be judged on the grounds of the usefulness of his work.

Hardy takes considerable pains to make a clear distinction between real mathematics and what he terms 'trivial' mathematics, and that, in rather superficial terms, the latter proves to be more useful than the former. Hardy then moves on to the discussion to which he had referred earlier in the book of whether mathematics, regardless of whether it is 'real' or not, can do harm. Unavoidably, the application of math during times of war

is the basis of this argument, though Hardy says real mathematicians can glean a modicum of comfort from the fact that 'real' mathematics has no application in war, at least not at the time of the writing of the book. Trivial mathematics, however, has considerable use across a broad range of disciplines, be it gunnery or aeronautics.

Hardy speaks of his childhood influences, one being a book by Alan St. Aubyn called *A Fellow of Trinity*, another being Professor Love, and another still was Jordan's *Cours d' analyse,* and Hardy says he cannot remember having ever wanted to be anything but a mathematician.

Hardy's life, he goes on to say, is finished and that at that point, i.e. at the time he wrote this book, nothing he could do would perceptibly increase or decrease the value of his life. If he had been offered a life, neither better nor worse, at the age of twenty, he says he would have accepted it without hesitation. Hardy also says that if success is the criterion on which a life is judged, he was right to be a mathematician.

Hardy considers nothing he has done to have been 'useful' and that so judged, the value of his mathematical life is precisely nothing. Hardy's sole reason for justifying his life as having more meaning than that of another more futile one is that he has added something to knowledge, and helped others to add still more. In some way the things which he has added are on a level commensurate with other great mathematicians and artists who have left something behind them.

Analysis

The tone of the novel is set incontrovertibly with the opening line: "It is a melancholy experience for a professional mathematician to find himself writing about mathematics." This sentiment pervades much of the text: Hardy has come to the realization that he no longer possesses the acuity of mind or thought to perform the function that has sustained him throughout his life and this, obviously, depresses him.

The scope of Hardy's exposition, whether he intended it this way or not, extends beyond the fundamental discussion of mathematics and its importance in his own life. His musings are a reflection of the state of mind that afflicts many people when they come toward the end of their life. Hardy questions his existence and consequently the meaning of his life. Hardy contemplates whether the world has been a more meaningful place for him having been in it, all questions that have plagued the minds of philosophers, artists, writers, and poets for centuries. Hardy considers mathematicians to be included in the numbers of these artists.

Creativity, Hardy says, is the most laudable of virtues. In fact, despite the pervasive sense of sadness in the work, Hardy's passion, even in light of his morbid state of mind, shines through. Hardy's belief in and appreciation of the beauty of mathematics is evident, and so is the esteem he holds for the creative minds of poets and writers. Early in the book, Hardy establishes the reasons why he chose to dedicate his life to mathematics, and thus, it becomes clear as the book unfolds that stripped of this

creative outlet, essentially an innate part of his being, Hardy feels as though he has lost a part of himself.

While much of *A Mathematician's Apology* is self-effacing, Hardy does display a modicum of pride at his association with Littlewood and Ramanujan, the latter of whom Hardy discovered, and despite his many assertions about the lack of utility of real mathematicians, he states that he would choose the same life again. By the end of the novel, Hardy finds justification for his existence, for what could be more of a positive indictment of one's life than the fact that, in hindsight, one would choose to do it all over again?

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Characters

Niels Henrik Abel

Niels Henrik Abel was a Norwegian mathematician (1802-1829) known for the tremendous amount of brilliant work he completed in his brief, twentysix- year life.

Francis Herbert Bradley

Metaphysician and philosopher F. H. Bradley (1846-1924) is most noted for his *Appearance and Reality* (1893), which was considered an important philosophical discussion of contemporary metaphysical thought at the time of its publication. He was also known by the influence his writing had on author T. S. Eliot.

Albert Einstein

German-born physicist Albert Einstein (1879- 1955) is considered one of the most brilliant men who ever lived. His theory of relativity, which he introduced in 1915, was revolutionary. It related matter with energy and displaced Newtonian mechanics as the cornerstone of physics by introducing the concept of space-time. In 1921, he received the Nobel Prize. A Jewish pacifist, Einstein immigrated to the United States shortly after Hitler came to power.

Euclid

Euclid of Alexandria, Egypt (approximately 325-265 B.C.), is the most prominent mathematician of antiquity, best known for his treatise on mathematics titled *The Elements*. Euclid taught in Alexandria, but little else is known of his life. However, the timelessness of *The Elements* has made Euclid the leading mathematics teachers of all time.

John Burdon Sanderson Haldane

British geneticist, biologist, and writer John Burdon Sanderson (J. B. S.) Haldane (1892-1964) was one of the most influential scientists of the early twentieth century and was well known for his left-leaning politics. His numerous works include *Callinicus: A Defense of Chemical Warfare* (1925). Disillusioned with the state of Marxism after World War II, he eventually moved to India, where he continued to conduct scientific research.

Lancelot Hogben

Educated at Trinity College, Cambridge, Lancelot Hogben (1895-1975) is best known for his book *Mathematics for the Million*, which is considered a classic in its field. In the book, Hogben covers the entire spectrum of applied mathematics from simple math to advanced calculus. The work received positive reviews from the likes of H. G. Wells and Albert Einstein, though it achieved prominence due to the harsh criticism it received from G. H. Hardy.

Alfred Edward Houseman

British poet Alfred Edward (A. E.) Houseman (1859-1936) was known for the argument made in his 1933 lecture "The Name and Nature of Poetry" that poetry should appeal more to emotions than to intellect. He published several collections, including *Last Poems* (1922). His final collection, *More Poems*, was published shortly after his death in 1936.

Dr. Samuel Johnson

The leading literary scholar and critic of his time, Dr. Samuel Johnson (1709-1784) was equally celebrated for his brilliant and witty conversation. The work that firmly established Johnson's reputation was his *Dictionary of the English Language* (1755), the first comprehensive lexicographical work on English ever undertaken.

John Edensor Littlewood

For thirty-five years, John Edensor Littlewood (1885-1977) collaborated with G. H. Hardy, working on the theory of series, the Riemann zeta function, inequalities, and the theory of functions. The collaboration led to a series of papers, *Partito numerorum*, using the Hardy-Littlewood-Ramanujan analytical method. Among his many awards, Littlewood was elected a fellow of the Royal Society in 1915 and received the Royal Medal of the society in 1929. He received the Sylvester Medal of the society in 1943.

Sir Isaac Newton

Sir Isaac Newton (1642-1727) was one of history's most influential and famous scientists. His work as a mathematician, physicist, and astronomer brought him world renown. Newton rings, Newton's law, and the MKS unit of pressure (the Newton) are named after him.

Pythagoras

Greek philosopher and mathematician Pythagoras (c. 580-400 B.C.) lived most of his life in Crotona, in southern Italy. His doctrines "Kosmos," "Metempsychosis," and the "Music of the Spheres" are well known. The famous Pythagorean theorem, concerning right-angled triangles, holds that the square of the hypotenuse (i.e., the long line opposite the right angle) is equal to the sum of the squares of the other two sides. This idea was current for many centuries beforehand, but Pythagoras was the first to prove it to be true.

Srinivasa Ramanujan

Srinivasa Ramanujan (1887-1920) was a celebrated Indian mathematician. He is well known for his contributions to the analytical theory of numbers, elliptic functions, continued fractions, and infinite series. Despite a lack of formal higher education and a life of ill health and severe poverty, Ramanujan proved to be a preeminent mathematical genius of his time. In 1913, he sent a paper to G. H. Hardy, who immediately saw his genius and arranged to have him take a position at Trinity College, Cambridge, where for the next four years the two men collaborated on what are considered to be five of the most remarkable papers in their field. In 1918, an impressive list of mathematicians proposed his name for election as a fellow to the prestigious Royal Society of London, a rare honor that was immediately bestowed upon him. Even after his death at the young age of thirty-two, his notes continued to be a subject of research and a source of further mathematical theorems, formulas, and solutions.

Bertrand Arthur Williams Russell

Bertrand Arthur Williams Russell (1872-1970) is considered one of the founders of analytic philosophy. In 1900, following his education at Trinity College, Cambridge, Russell became acquainted with the work of Italian mathematician Giuseppe Peano. Peano's work inspired him to write *The Principles of Mathematics* (1903), which he subsequently expanded in collaboration with Alfred North Whitehead into the three volumes of *Principia Mathematica* (1910-1913). Russell's many essays, often in the form of short reflections or observations on moral or psychological topics, are written in a terse, vivid, and provocative style. Russell was also well known for his pacifist views, which cost him his job at Cambridge during World War I and also brought him a six-month jail sentence. His greatest literary achievement was *A History of Western Philosophy (1945)*.

Charles Percy Snow

Charles Percy (C. P.) Snow (1905-1980) led a varied career that included scientific and civil service work, but he is best known as the author of the serialized fictional work entitled *Strangers and Brothers* (1940). His schooling was in chemistry and physics, and

during World War II he served as director of technical personnel for Britain's Ministry of Labour. In 1957, he was knighted, and in 1964 he was named baron for his services to the Ministry of Labour. His 1959 Rede Lecture on "The Two Cultures and the Scientific Revolution," lamenting the increasing gulf between "literary intellectuals" and "scientists," provoked widespread and heated debate. In addition to his work in the sciences, Snow was the author of much short fiction published by *London's Sunday Times*, and over the course of his lifetime he published more than a dozen novels.

Themes

Aesthetics

One of Hardy's principal arguments is that theoretical mathematics, which he refers to as "real" or "pure" mathematics, has similar aesthetic qualities to those of art or poetry. Hardy invests much in his essay defending this position, explaining the beauty of Pythagoras's and Euclid's theorems, and comparing the aesthetics of pure mathematics to the simplistic and vulgar exercises that make up applied mathematics.

Aging, Prime of Life, Depression, and Melancholy

A Mathematician's Apology was written during the final years of Hardy's life, shortly after a heart attack and a series of other physical ailments had rendered him mostly sedentary. This theme colors much of the text. Whereas in his prime he could devote his days to intense studies of concepts and vigorous games of cricket, those abilities were long lost to him as he was writing this memoir. Hardy firmly believed that mathematics is a young man's game. He uses several mathematicians—including Ramanujan, Newton, and others—as examples of geniuses who peaked in their twenties and thirties. By the time of the writing of this memoir, Hardy was in his sixties. This resulted in a melancholic tone that borders on depression. A few years following the publication of the book, Hardy unsuccessfully attempted suicide by taking an overdose of barbiturates.

Creative Process

Throughout *A Mathematician's Apology*, Hardy compares the "real" mathematician to the creative artist. He uses poetry and art to make this comparison. He believes there is an objective "mathematical reality" that exists in the world, which is no different from the "physical reality," and it is up to the mathematician to discover and describe that reality. The best of pure math can be held as the highest of all art forms.

Genius, Common Man/Everyman

At the expense of being criticized for elitism and snobbery, Hardy distinguishes between those who can perform a single task adequately—of which there are a small minority—and those who can perform a single task in their lives exceptionally, of which there are a significant few. These are the geniuses of the world, and Hardy is proud to have worked alongside those men he considered to be the most ingenious of all time, including Ramanujan and Littlewood.

Self-Doubt

Despite Hardy's elitist tendencies and tremendous confidence in his own intellectual abilities and importance, *A Mathematician's Apology* is imbued throughout with Hardy's severe self-doubts about his own worth as a human being and the worth of his contributions to mathematics and to the world. These self-doubts were, undoubtedly, caused in large part by his deteriorating physical state at the time of his writing and also in large part by the accumulation of years of criticism he received for so many of his views. The effects of years of being a "misunderstood genius" appear to have taken their toll, and one of the underlying purposes of writing this memoir is for Hardy to determine for himself if his life has been worthwhile.

Superiority, Egotism/Narcissism, Vanity, Conceit

Hardy admits that *A Mathematician's Apology* is an egotistical work. Men—and here Hardy includes himself—who choose to make a career out of mathematics do so in order to achieve a certain status of immortality. And if they sit down to write about their lives or their work, they do so because of their conviction that they have done something remarkable and should be remembered for it. Hardy does not hold back from stating his belief that he has made significant contributions to his field and that he is among the elite of the world in his field. Despite these views, however, his work embraces a certain amount of humility in that he recognizes greater geniuses than himself, and he feels proud to have considered them to be among his colleagues and friends.

War

Hardy's famous collaboration with Ramanujan occurred during World War I, a war which Hardy adamantly opposed for both philosophical and practical reasons. Unlike most of his colleagues, Hardy held German society in high regard due to its advances in scientific thought, and he seriously mistrusted the British politicians. As a result, he was one of the few distinguished thinkers of his day, along with Bertrand Russell, who refused to support the war. On a practical level, Hardy thrived through his collaborations, many of which were with colleagues throughout Europe. The war had a tremendously disruptive influence on these efforts and hampered his professional development.

One of the great ironies of *A Mathematician's Apology*, written on the eve of World War II, is that Hardy defends the ethics of pure mathematics on the grounds that it is a "gentle and clean" field of study, unlike its counterpart, applied mathematics, which can make claim to its contributions, for instance, to the fields of ballistics. Hardy refuses to admit, or is unable to see, a causal relationship between theoretical math and warfare. He even goes so far as to predict that it would be years before Einstein's theory of relativity could be applied to any real-life situation.

Style

Apology

A Mathematician's Apology is, as the title implies, written in the form of an "apology," or defense. In this case, the author sets out to defend his chosen career: namely, theoretical, or what he calls "pure," mathematics. Although he was generally accepted for his brilliant theoretical insights, which resulted in many remarkable works and collaborations, Hardy's view that theoretical mathematics is an art form, while its counterpart, applied mathematics, is at best an application of trivial exercises, caused great disagreement among his contemporaries and thus spurred the need for this defense.

Tone

With this book, Hardy set out to address a general audience of both mathematicians and nonmathematicians alike, and as a result he employs a narrative style that could best explain in simple terms his profound and complex array of ideas. To that end, his tone, while often conveying a derogatory and elitist attitude toward his subject matter, never condescends to the reader with lofty diction; anyone with a rudimentary knowledge of mathematics would feel at home and comfortable with Hardy's style. At the same time, the ideas he expresses are of a depth that would satisfy his colleagues.

Hardy himself is an archetype of the misunderstood artist; a creative genius who was either far ahead of his times or hopelessly behind. As history has proven, he was a little of both. Hardy's own refusal to bow to the conventions of the time in regards to any subject matter, and his irrepressible need to offer his opinions and ideas regardless of the potential social or professional consequences, placed him in this lonely position. As a result, *A Mathematician's Apology* is anything but objective. While Hardy's argument is generally well defended, many of his subjective views, especially in regards to applied mathematics and chess, have been harshly criticized, and it is clear that as his life was drawing to a close, Hardly had achieved a melancholic acceptance of this position. Nevertheless, even to the end he refused to retreat on any of the views that defined his life and career.

Historical Context

Since Hardy elucidated a philosophy that stresses the timelessness and immortality of pure mathematics, it is very difficult to contextualize *A Mathematician's Apology* with respect to a single historical period. Viewed as an autobiographical memoir, *A Mathematician's Apology* is a product of a genius who came of age towards the end of the Victorian era and who died as the world entered the nuclear age. However, viewed as a philosophical treatise and justification of mathematics, the book begins with the ancient Greeks and extends to the eve of World War II.

Hardy was a product of the English educational system that retained intellectually mediocre clergymen as the main instructors until well into the nineteenth century. Although this system had been largely reformed by the turn of the century, mathematics was one of the last disciplines to be affected. Cambridge mathematician Norbert Wiener described the level of mathematics at Oxford as "contemptible." The training available at Cambridge was not much better, consisting of a severe exam system, the triposes, which relied on rote memorization rather than any degree of unique creativity; it was not a system that inspired a mathematical genius. The insufficiency of the English system meant that the English lagged behind other European countries in producing mathematicians and modern mathematical theories.

In his memoir, Hardy emphasizes that he became a mathematician in spite of this early training. Much of *A Mathematician's Apology* can be read as a subtle jab at the stifling environment of the English educational system. As Hardy's generation of mathematicians gained international recognition, they quickly instituted changes that treated mathematics more as a creative art than as an endless series of exercises in rote learning. Hardy himself, for instance, was instrumental in opposing the continuation of the rigid tripos exam system.

During the early part of the twentieth century, Britain was still very much an empire with territories spanning the globe, including India. The great Indian mathematician Srinivasa Ramanujan learned English as a result of the English colonial system. It was as a result of Britain's close relationship with India that Ramanujan and Hardy began a correspondence which was to result in one of the great collaborations in mathematics history. Hardy was able to procure a position for Ramanujan at Trinity College, Cambridge, which allowed the collaboration to flourish.

But Hardy's other collaborations at the time did not fare so well. By the time World War I broke out, Hardy was in his prime and had already begun working with several other mathematicians outside of England, who would ultimately have a lasting effect on both his own career and on mathematics as a whole. But with the outbreak of the war and the virulent nationalism that accompanied it, international collaboration proved exceedingly difficult, if not outright impossible.

Hardy became well known throughout his life for his outspoken views outside of the field of mathematics. The narrow-mindedness of many scientists within England, for

instance, greatly concerned Hardy, and he let that be known. Unlike most intellects of his day, Hardy had a great reverence for the German mathematical school and was greatly distressed by the anti-German sentiment that proliferated throughout England and particularly at Cambridge. His ability to separate German intellectual achievement from the exaggerated "inhuman" traits of the enemy which were spoken of throughout England made him somewhat of a pariah figure in this regard. He even went so far as to carry on an extensive correspondence with Swedish mathematician Gösta Mittag-Leffler, in which the two worked towards a reconciliation between German and Allied mathematicians with the war still raging.

Hardy wrote *A Mathematician's Apology* under the threat of another world war. Although he could not ignore the threat of that war, it is almost as if he includes the relationship between war and mathematics as an afterthought. He admits this in a brief note that follows the last chapter. Viewed with the hindsight of today, his views concerning the improbability that a theory like relativity would have an effect on war in Hardy's lifetime appear to be grossly miscalculated and anachronistic. These views are the last gasp of an age of innocence and naiveté, ignoring or not fully recognizing the devastating effects that science—even the science of pure mathematics which Hardy considered to be "gentle and clean"—could have on humanity.

Critical Overview

In his review of *A Mathematician's Apology* in the *Spectator*, British author Graham Greene asserts that Hardy's philosophy is akin to the philosophy of an artist. "The real mathematician," according to Greene, "must justify himself as an artist." Indeed, Hardy's work is a very successful justification of the mathematician as artist, much in the literary tradition that includes *The Autobiography of Benvenuto Cellini*; Vincent Van Gogh's letters to his brother Theo; and, as Greene points out, the work of Henry James. Greene writes, "I know no writing—except perhaps Henry James's introductory essays—which conveys so clearly and with such an absence of fuss the excitement of the creative artist."

While the "uninitiated"—that is, non-mathematicians such as Greene—were apt to focus on the work as an artist's memoir, those with more rigorous mathematical training focused on the rift within the field of mathematics that *A Mathematician's Apology* brought to the fore. As the anonymous reviewer in the *Times Literary Supplement* observes, "'Real' mathematics deals only with the ultimate abstractions of number, and, if not in itself incapable of being put to 'use,' at least becomes only occasionally and accidentally useful." "Applied" mathematics, on the other hand, deals with numbers as useful scientific tools, which helps bring about innovation. Its definition implies utility, or usefulness, and is the opposite of the "math-as-art philosophy" Hardy espouses throughout the book. And true to Hardy's lifelong reputation for his candid opinions, Hardy did not hold back the scorn and derision he felt for the functional uses of mathematics. He refers to chess problems, for instance, as "trivial," regardless of their relative degrees of difficulty, and he similarly belittles applied mathematicians and their work throughout the book.

Hardy sums up this attitude at the beginning of chapter 28:

There are then two mathematics. There is the real mathematics of the real mathematicians, and there is what I call the "trivial" mathematics, for want of a better word. The trivial mathematics may be justified by arguments that would appeal to [Lancelot] Hogben, or other writers of his school, but there is no such defense for the real mathematics, which must be justified as art if it can be justified at all.

Ironically, Hogben, the mathematician for whom Hardy reserved the word "trivial," appears to have been unaffected by the criticism. In fact, a late edition of Hogben's book, *Mathematics for the Million*, was reviewed in tandem with the reprint of *A Mathematician's Apology* in 1967 in the *Times Literary Supplement*, as a vivid illustration of the disagreement between the two views. As the reviewer notes, "For [Hardy] Hogben is 'admittedly not a mathematician' and 'real' mathematics is to Hogben 'merely an object of contemptuous pity.'" Despite the profound differences between the two works, the reviewer writes that they both "deserve the immortality they appear to have achieved."

Criticism

- Critical Essay #1
- Critical Essay #2

Critical Essay #1

Partikian is a freelance writer, editor, and English instructor. In this essay, Partikian suggests that Hardy's text is a multifaceted work that should be appreciated primarily as an artistic treatise and memoir.

Mathematics is an exclusive club that opens its doors to a small number of gifted and often misunderstood individuals. Those who remain outside only have a vague perception of what it means to be a mathematician, and the perception that they do hold is more often than not hindered by an inability to understand exactly what it is that a mathematician does. Conversely, the expert mathematician is almost as ill-equipped as the layperson in trying to convey the beauty and joy of pure mathematics to non-mathematicians; the mathematician's reliance on abstractions and the specialized vocabulary that define him as a mathematician make him a poor choice to describe and verbalize his field to the layperson. One either grasps the inherent beauty of theorems and numbers, thereby earning entry into the club, or one cannot and remains a perplexed outsider unable to grasp the inscrutable formulas. At least this was the case until G. H. Hardy, one of the foremost mathematicians of the twentieth century, bridged the gap and allowed the non-mathematicians of the world a glimpse into the mind and values of a pure mathematician.

Past his intellectual prime and restricted physically by several years of failing health, Hardy decided to write *A Mathematician's Apology*, a book that can be appreciated by the mathematician and non-mathematician alike. The theorems that he outlines are among the most basic in the entire field. They are chosen so that the reader can both readily comprehend the explanations and easily perceive their aesthetic qualities. Hardy writes with the flavor and passion of an art lover about Euclid's proof of the existence of an infinity of prime numbers and Pythagoras's proof of the irrationality of the square root of two. The theorems he describes are representative of works of art precisely because they are so simple, which also makes them convenient as perfect examples for the general reader.

Since Hardy writes for an audience in large part comprising non-mathematicians, one must classify his essay with literary rather than mathematical headings. While the cold narrative voice of a mathematician does come forth at times in the prose, so do tones of elitism, disdain, and artistic snobbery, all of which do not normally belong in a mathematical essay. How then should we classify the essay if not as "mathematical"? *A Mathematician's Apology* is so multifaceted that it seems to transcend pigeonholing or categorizing. Restricting it to any single genre is an error that would cause a very restricted interpretation. For a thorough understanding of Hardy's intentions, one must read the work as a representative example of various literary genres including the apology, artistic manifesto, and memoir. In each of these genres, Hardy's elitist, valueladen tone invariably either demands unconditional acceptance or provokes severe disagreement.

A literary apology is a defense or justification for a particular way of life. Hardy, in calling his essay an apology, feels compelled to defend his chosen discipline. The urgency which he brings to the task leads one to believe that he is, at any given moment, trying to convince himself of the arguments. In order to present his belief that mathematics is an art, Hardy returns again and again to the concept of "utility" or "usefulness." Judging from the disproportionate amount of writing he dedicates to these definitions, the charge that real mathematics has no practical use must have truly bothered him over the years. The disdain Hardy reserves for the widely accepted notions of "utility" and "usefulness" is a further indication that the core of Hardy's philosophy resides in these definitions.

Hardy continually splits hairs in defining "utility" or "usefulness" in order to refine a definition that contradicts common sense. After Hardy has finished, the conventional conceptions of "useful" and "useless" have been inverted from what is generally accepted: what we commonly hold to be "useful" applications, such as engineering, geometry, and calculus, are "trivial" and useless to the real mathematician, according to Hardy. Conversely, the "uselessness" of real mathematics is precisely the reason why it is immortal and why one can consider it an art form. Hardy cannot contain his contempt and scorn for applied mathematics, calling it "school" mathematics and referring to its various worldly applications—such as engineering feats, ballistics, and aerodynamics—as "repulsively ugly and intolerably dull."

In belittling an entire subfield of mathematics, Hardy puts himself in a difficult position from which he can only extricate himself by twisting conventional definitions to justify his own field. His disdain for practical and useful applications forces him to redefine "uselessness," a word that usually evokes negative images, in a manner that brings forth positive connotations. Herein lies the second stage of his definition and the beauty of the essay. Hardy may appear, to the careless reader, to have painted himself into a corner by proclaiming that it is "not possible to justify the life of any genuine mathematician on the ground of the 'utility' of his work." According to Hardy's philosophy, "applied" mathematics is "trivial" because it is useful, while "real" mathematics is immortal and superior because it is useless. Furthermore, it is useless in the way that the highest art forms of humankind are useless. Much the way Taoist thought holds a certain type of uselessness as an outstanding character trait, Hardy compares the uselessness of "real" mathematics to the uselessness of art. In this sense, to be useless is the ultimate compliment, and "real" mathematics is the highest form of art. Hardy writes

For mathematics is, of all the arts and sciences, the most austere, and the most remote, and a mathematician should be of all men the one who can most easily take refuge where, as Bertrand Russell says, "one at least of our nobler impulses can best escape from the dreary exile of the actual world." . . . Mathematics is not a contemplative but a creative subject.

In setting down this philosophy and carefully describing its terms, Hardy creates a manifesto that describes real mathematics as an artistic movement, in much the same way the surrealist André Breton clarified an artistic movement in his *Manifesto of Surrealism*. Hardy puts forth the argument that real mathematicians have since time

immemorial been artists of the highest caliber. This mathematicianas- artist motif was noticed immediately in early reviews of the work. In his 1941 review of *A Mathematician's Apology* in the *Spectator*, British author Graham Greene asserts that Hardy's philosophy is akin to the philosophy of an artist. "The real mathematician . . . must justify himself as an artist," Greene writes.

Putting all accolades aside, there are those who remain unconvinced of the basic theories in Hardy's concept of mathematics as art and who take offense to his dismissive views of other artistic genres. Arthur Waley writes in an early review for the *New Statesman* and *Nation* that "Dr. Hardy in this book is very definitely on the defensive, and his defense of mathematics consists in asserting that it is an art, like painting or poetry." However, a poet could easily take offense or pick apart the examples and arguments that Hardy puts forth in an attempt to show the inferiority of poetry as an art. Not only does Waley not buy into the logic, he writes, "All this sounds like the comment of one whose contact with poetry is somewhat superficial." A similar argument can be proposed for many of the other disciplines and fields Hardy writes off as "trivial." The most glaring example is chess. Chess grandmaster Alexander Alekhine is derisively described as a "conjuror" or "ventriloquist," and chess is constantly belittled as "trivial." Keeping Waley's objection in mind, it is clear that Hardy knows no more about chess than he does about poetry. His analysis and dismissals are superficial in that they do not take into account, for instance, the countless variations of set openings and the economy and aesthetic beauty of eliminating inefficient continuations in an attempt to bring about a "winning" position. In short, much as with poetry, Hardy writes off an entire field or artistic genre as inferior without having approached it with the same passion and knowledge he retains for theoretical mathematics; a chess grandmaster could easily argue that some of José Capablanca's games contain the same simplicity and beauty of a Euclidian theorem.

Hardy's derisive tone does not in any way imply final authority. The mathematician Lancelot Hogben (Hardy hesitates to even confer the title of mathematician on him), for whom Hardy reserves a flagrant contempt, has also achieved a significant and enduring reputation. A reprinted version of Hogben's book *Mathematics for the Million* was reviewed in tandem with the reprint of *A Mathematician's Apology* in 1967. The continuing popularity of both works indicates a particular rift within the field of mathematics and clearly shows that Hardy, though universally accepted as brilliant, is not necessarily considered the final authority he claims to be.

Although Hardy's artistic philosophy has provoked widespread disagreement, his work remains extremely compelling as a personal memoir. Snow, writing in his biographical portrait of Hardy that initially appeared in his *Variety of Men* and that is now included as the introduction in later editions of Hardy's essay, believes it to be a work of "haunting sadness" precisely because it is a "passionate lament for creative powers that used to be and that will never come again." Viewed as a memoir, the work, particularly towards its conclusion, describes the trials and tribulations that a creative genius must undergo to excel in a field that does not appear to have the approval of the non-initiated. Both Snow's biographical portrait and Hardy's concluding chapters, for example, mention the insufficiency and stifling quality of the English educational system to which Hardy was

subjected during his formative years. The mathematics departments at Cambridge relied on a contemptible and severe exam system, the triposes, which rewarded diligence rather than creativity. Hardy became a mathematician in spite of his education and was never truly appreciated by that system for the creative thinker that he was. The artistic genius is bound to remain misunderstood and held back by a callous society of Philistines; such a theme appears in numerous artistic memoirs and biographies of creative thinkers.

Although past the prime of his ability to contribute to the field of theoretical mathematics, Hardy retained the ability to describe to the layperson why the field of real mathematics is so lofty and noble. His eloquence and reluctant acceptance of his declining abilities allowed him to bridge the gap between genius and the common person, leaving a unique memoir to accompany his more creative artistic and mathematical endeavors. And although his derogatory statements and biased appraisal of real mathematics as the loftiest art form make him appear irrepressibly elitist, an undertone of humility caused by the realization of his declining physical and intellectual abilities balances Hardy's writing and has rendered *A Mathematician's Apology* an enduring classic.

Critical Essay #2

Holm is a freelance writer with speculative fiction and nonfiction publications. In this essay, Holm discusses the parallel Hardy draws between the pure mathematician and the artist.

Mathematics may not be the first pursuit that comes to mind when we speak of the creative process. The artist and the mathematician may seem to be on different ends of the spectrum. Storytelling, painting, literature, dance—these appear to be the realm of creative artists. Math, on the other hand, is an "austere" profession, little understood and sometimes feared. In *A Mathematician's Apology*, G. H. Hardy distinguishes between pure and applied mathematics and compares the pursuit of pure mathematics to the creative process. For the most part, the comparison works.

According to its definition, the word "create" means to bring into being, to make, or to make by giving a new character function or status. Creation is the formulation of the new. Writers create stories that have never been told or have never been told with that author's particular slant. Musical composers create original works or variations on existing works. Visual artists bring their visions into being using a number of media. And so, claims Hardy, pure mathematicians create new thought and new direction with their medium—numbers. In the introduction to *A Mathematician's Apology*, C. P. Snow refers to a review of Hardy's book by Graham Greene; Greene called *A Mathematician's Apology* one of the best accounts of "what it was like to be a *creative artist*."

Snow, who knew Hardy personally, claims that Hardy had little "ego" and thus had to make a great effort in later life to assert his opinions. According to Snow, this aspect of Hardy's personality contributed to the "introspective insight and beautiful candour" of Hardy's thought process and writing. Throughout the book, Hardy clearly displays an artist's passion for his work, with candor that can be direct and unflinching. Hardy discusses the creative artist's potential to do something exceptional, and with characteristic bluntness and high standards claims that "if a man has any genuine talent, he should be ready to make almost any sacrifice in order to cultivate it to the full."

While Hardy promotes these standards for those with talent, he writes off much of the human population by claiming that "most people can do nothing well at all." For the reader who never knew Hardy personally, it is hard to tell whether this remark is indicative of excessive ego or of a creative person's high demands of himself and what he wishes to accomplish in life. To Snow, at least, Hardy's purpose in life and in the field of mathematics was "to bring rigour into English mathematical analysis." Hardy's purpose is so integral to his existence that he admits this and calls it "inevitable egotism." Good work, says Hardy, "is not done by 'humble' men." Like a creative artist, Hardy believes that for a human, "the noblest ambition is that of leaving behind one something of value."

Throughout *A Mathematician's Apology*, Hardy does not deny that he accomplished his life goal of bringing rigor into his field. However, he bemoans the fact (or the perception)

that mathematicians do their best, most groundbreaking work at a relatively young age. The cause of Hardy's sadness is one aspect of a mathematician's life that seems to deviate from that of some creative artists. Hardy never tells the reader why older mathematicians do less than cutting-edge work. We might assume that as mathematicians age, their mental faculties decrease. However, creative artists in certain other fields may paint, write, or perform well into old age. Given the creative artist's passion for creating, the reader might correctly assume that being unable to continue to create is akin to personal catastrophe or unbearable sadness.

It would be interesting to know how much satisfaction Hardy gleaned from continuing to work in mathematics into older age. Was his work truly inferior to what he had produced at a younger age? Was he able to feel as passionately about his later work in the field? In some creative endeavors such as literature, age, maturity, and experience may enrich the final product. An author's first novel published when she is twenty-five is likely to be vastly different than a novel the same author publishes when she is forty-five. Snow concurs with this view of literature, stating that "it is very rare for a writer to realize, with the finality of truth, that he is absolutely finished." But Snow, like Hardy, never explains why the work or the art of aging mathematicians diminishes.

Hardy's world of pure mathematics in this respect more resembles the career of an athlete or a dancer. Why is pure mathematics such an "all or nothing" proposition? Why does Hardy believe that when a creative man has lost the power or desire to create, "it is a pity but in that case he does not matter a great deal anyway, and it would be silly to bother about him?" Has Hardy really lost the ability to continue to create or is he feeling the pinch of competition from up-and-coming, younger mathematicians? While competition may enter into an artist's life, it does not need to affect the ability to continue to produce.

Regardless, it is Hardy's exposition of the mathematical process as a creative process that makes *A Mathematician's Apology* so accessible to the non-mathematical reader. Certainly, readers who are involved in some form of artistic creation or readers who have passionately and single-mindedly pursued the creation of something new in their lives are shown mathematics in a new light. And this is a good thing. Hardy admits that many people have an irrational fear of basic, applied mathematics. How could such people, therefore, be expected to understand the esoteric realms of pure mathematics, a field which Hardy calls "the most austere and most remote of all the arts and sciences"?

To Hardy, artists as well as mathematicians create patterns. Like the patterns that a poet or painter creates, the patterns that the mathematician creates must be beautiful. Hardy claims that the ideas in any of these forms need to flow well together; "there is no permanent place in the world for ugly mathematics." On the other hand, Hardy readily admits to the difficulty of defining beauty, a dilemma shared by artists and mathematicians alike. Worthwhile mathematics, according to Hardy, should be "serious as well as beautiful—'important.'"

One parallel between the creative process and the study of pure mathematics that Hardy does not elaborate upon is the role of the unknown. During the process of creativity, depending upon the particular artist and his or her style or mode of work, the end result may be completely unknown. For some creative artists, this is part of the thrill of creation. Hardy never alludes directly to any personal fascination, distaste, or indifference to this aspect of the creative process. We can probably assume that most mathematicians would be thrilled, in the course of their work, to discover something previously unknown; something so cutting-edge that it changed the direction of the field of mathematics and had profound implications. But the role of the unknown in the creative process can take on subtler aspects. In the area of novel writing, for example, some authors outline a novel completely before they start to write. Other novelists refuse to outline, writing the novel and figuring out the story, plot, and ending as they go. Many authors fall somewhere in between on this spectrum; outlining and preplanning to some degree but becoming fluid if needed to change course. While it may not be fair to compare pure mathematics research to novel writing, it might be interesting to know how comfortable or uncomfortable Hardy was with the unknown during his own research processes.

Hardy certainly has the purity of vision of a creative artist, or of anyone who knows what he wants to do and is doing what he loves. Immediately, he makes it clear that he prefers to do mathematics rather than engage in "exposition, criticism, appreciation-work for second-rate minds." Critics, to Hardy, rank lower than scholars or poets, and he admits that it is a confession of weakness on his part to write about mathematics rather than actually writing mathematics. Like a creative artist, Hardy is so sure of his passion for his subject that "a defence of mathematics will be a defence of myself." The artist and the art seem to be one and the same.

Finally, Hardy makes clear the difference between applied and pure (or real) mathematics, and it is clear that his heart and work are in the latter. To Hardy, the position of an applied mathematician is

in some ways a little pathetic . . . he wants to be useful, he must work in a humdrum way, and he cannot give full play to his fancy, even when he wishes to rise to the heights. "Imaginary" universes are so much more beautiful than this stupidly constructed real one."

As a creative artist might, Hardy sees value in transcending the "real" in pursuit of creativity, beauty, and significance. He can see no other way to justify real mathematics, other than justifying it as art, a view he claims is common among mathematicians.

Source: Catherine Dybiec Holm, Critical Essay on *A Mathematician's Apology*, in *Nonfiction Classics for Students*, Gale, 2003.

Topics for Further Study

The ethical issues surrounding theoretical research are complex. Hardy called pure mathematics "gentle and clean." However, Einstein's theory of relativity played a direct influence in the development of the first atomic bomb. Research the role Einstein actually played in the development of the atomic bomb. Discuss the letter to President Franklin Delano Roosevelt that he signed urging the president to expedite its development in order to stop the spread of Nazism. How can you reconcile this act to Einstein's later pacifism?

While people often read about the antiwar movement during the Vietnam War, one seldom hears about antiwar sentiment during World War I or World War II. In fact, Hardy and Bertrand Russell were among the few intellectuals of their day to speak out openly against the wars. Russell was, in fact, jailed briefly on account of his beliefs and activities. Research and write an essay about the antiwar movement during the First and Second World Wars, with an emphasis on the role intellectuals played. Include the roles Hardy and Russell played.

Hardy believes that ancient mathematicians will be remembered for their influence long after their counterparts in the arts, literature, and philosophy are forgotten. He mentions Euclid and Pythagoras as two of the classical mathematicians who have achieved immortality. What other mathematicians from ancient civilizations have achieved similar status? Do you believe that their mathematical contributions exceed the contributions made by ancients of other disciplines, such as Plato, Socrates, and Homer? Why or why not?

Among Hardy's proudest achievements, and what he is best remembered for, are his thiry-fiveyear collaboration with John Edensor Littlewood and his collaboration during World War I with Srinivasa Ramanujan. Outside of mathematics, there are numerous examples of famous collaborations. Research and write about one or two famous non-mathematical collaborations in the history of science or the arts.

Hardy is clearly a product of Victorian England, particularly of its educational system. Research the aesthetic values and the social and cultural mores of Victorian society. What kind of influence do you believe they played in Hardy's development?

Compare and Contrast

1930s: Technology still has limited, though powerful, uses in warfare; a war must be won largely from the strength of armed forces, with technology playing a secondary role.

Today: The United States uses advanced technologies in its bombing campaigns against Iraq and Afghanistan, thus severely limiting the need for ground troops directly engaging in battle.

1930s: The world has still not been exposed to the threat of nuclear annihilation. Nuclear fission is viewed as impractical, and Einstein's theory of relativity is still a concept remote from everyday life

Today: With the help of Einstein's theories, many nations have nuclear capabilities and can cause the destruction of mankind. Atomic testing in certain desert and ocean regions has had a lasting and adverse affect on the environment, and the threat of nuclear war between states continues to exist.

1930s: Alexander Alekine, Mikhail Botvinnik, and José Capablanca are celebrated for their mastery of chess. Chess is viewed as a game with an infinite number of continuations that can only be mastered by a particular kind of genius.

Today: Chess computers have been developed that can beat some of the best players in the world. IBM has developed a computer that defeats the reigning champion grandmaster, Gary Kasparov. Computers are capable of calculating an immense number of various chess continuations.

1930s: The profession of mathematics is an exclusive club, with a nearly all-male membership. In *A Mathematician's Apology*, Hardy does not mention a single female colleague or refer to a single female author.

Today: Although less than one-fifth of all mathematicians and scientists are female, the Association of Women in Mathematics, founded in 1971, has over 4100 members, and there is wider recognition that gender disparities in the field are an issue to be addressed.

What Do I Read Next?

Copenhagen (1998), a play by Michael Frayn, illustrates the moral issues faced by mathematicians and physicists during World War II. Hardy touches upon many similar subjects in his apology.

Mathematics for the Million, by Hardy's contemporary Lancelot Hogben, was originally published in 1937 and republished in 1967. It is an influential work in the field of mathematics and offers a sharp contrast to Hardy's view of applied mathematics as trivial.

Hardy's *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work* (1940) is inspired by Hardy's working relationship with famed Indian mathematician Srinivasa Ramanujan. This book is only for those wellversed in mathematics.

Harold Schonberg's *Grandmasters of Chess* (1973) provides brief biographical portraits of chess grandmasters as men of genius and artists, including Alexander Alekhine. These portraits contradict Hardy's assessment of chess and chess grandmasters as inartistic.

The MacTutor History of Mathematics Archive, compiled by the University of St. Andrews, Scotland, and found on the World Wide Web at http://www-history.mcs.st-andrews.ac.uk/ (last accessed December 2002), contains, among other useful mathematics history, an archive of more than 1300 mathematicians' biographies. Information on Hardy includes a bibliography and several articles written by and about him, along with several obituaries.

Sylvia Nasar's *A Beautiful Mind: The Life of Mathematical Genius and Nobel Laureate John Nash* (1998) is a biography of John Nash, Princeton professor and mathematician, whose primary work in Game Theory was already underpinning the economic system within his lifetime. Nasar's portrait of Nash's life includes sympathetic examinations of his work, his personal life, and his battle with schizophrenia.

Further Study

Berndt, Bruce C., and Robert A. Rankin, eds., *Ramanujan: Essays and Surveys*, American Mathematical Society, 2001.

This collection of largely non-technical, highly accessible essays on the Indian mathematician, is the first of three books covering Srinivasa Ramanujan's life and includes several articles on his wife, his Indian colleagues, and his long illness.

Chan, L. H., "Godfrey Harold Hardy (1877-1947)□the Man and the Mathematician," in *Menemui Matematik*, Vol. 1, 1979, pp. 1-13.

Chan provides a biographical portrait of Hardy that can be compared to that by C. P. Snow.

Golomb, Solomon W., "Mathematics after Forty Years of the Space Age," in *The Mathematical Intelligencer*, Fall 1999, p. 38.

Examining Hardy's assertion that pure mathematics has no relationship to issues of everyday life due to its inapplicability, Golomb argues that technological advances in the forty years since the publication of *A Mathematician's Apology* have largely proved his assertion to be false. Prime number theory, for instance, an area Hardy had a special claim to, has contributed to advances in cryptology. Golomb explains how several other mathematical fields that were also once considered "pure" are now clearly "applied," and he recounts his own experiences working in space programs during the fifties to further argue his case.

Hardy, G. H., Bertrand Russell's Trinity, Arno Press, 1977.

This book was originally published privately for Hardy by the University Press of Cambridge in 1942. In 1916, the philosopher Bertrand Russell was expelled from Trinity College, where he was lecturing, due to his objection to World War I. Hardy, who defended Russell and helped get him reinstated to the college after the war, sets out in this book to provide a full account of that incident and further helps to elucidate the lesser known history of conscientious objection during World War I.

Hoffman, Paul, *The Man Who Loved Only Numbers: The Story of Paul Erdös and the Search for Mathematical Truth*, Hyperion, 1998.

Hoffman provides a popular account of the life of the Hungarian mathematician Paul Erdös, who died in 1996 and was widely revered as one of the most prolific, if not the most bizarre, mathematicians who ever lived. He was known as much for his obsessive nature, nomadic existence, and boundless energy for the search for mathematical proofs as he was for his actual contributions to the science. Hoffman's book achieved widespread popular recognition at its publication and is accessible for the lay reader.

Kanigel, Robert, *The Man Who Knew Infinity: A Life of the Genius Ramanujan*, Scribner's, 1991.

Regarded as the definitive biography of Ramanujan, this book covers the mathematician's life from his early childhood to his death in 1920 with a strong emphasis on his years in England, where he collaborated with Hardy.

"A Professor's Ideals," in *Times Literary Supplement*, January 18, 1941, p. 33.

This article reiterates Hardy's philosophy that mathematics is a quest for beauty and truth.

Snow, C. P., *Variety of Men*, Scribner's, 1967.

C. P. Snow, a writer and a scientist who was a contemporary of Hardy, writes essays about several key early twentieth-century figures, including Hardy. The biographical sketch on Hardy has come to be included as the introduction in most modern editions of *A Mathematician's Apology*.

Wiener, Norbert, "Obituary: Godfrey Harold Hardy (1877-1947)," in *Bulletin of the American Mathematical Society*, Vol. 55, 1949, pp. 72-77.

This obituary gives an overview of Hardy's life and also details the problems he faced as a young man in the stifling English educational system.

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Greene, Graham, "The Austere Art," in *Spectator*, Vol. 165, December 20, 1940, p. 682.

"People Who Count," in *Times Literary Supplement*, December 28, 1967, p. 1266.

Snow, C. P., "Foreword," in *A Mathematician's Apology*, by G. H. Hardy, Cambridge University Press, 1967, originally published in *Variety of Men*, by C. P. Snow, Scribner's, 1967.

Waley, Arthur, "The Pattern of Mathematics," in *New Statesman* and *Nation*, Vol. 21, February 15, 1941, p. 169.

Wiener, Norbert, "Obituary: Godfrey Harold Hardy (1877-1947)," in *Bulletin of American Mathematics*, Vol. 55, 1949, pp. 72-77.

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Introduction

Purpose of the Book

The purpose of Nonfiction Classics for Students (NCfS) is to provide readers with a guide to understanding, enjoying, and studying novels by giving them easy access to information about the work. Part of Gale's□For Students□ Literature line, NCfS is specifically designed to meet the curricular needs of high school and undergraduate college students and their teachers, as well as the interests of general readers and researchers considering specific novels. While each volume contains entries on

□classic□ novels frequently studied in classrooms, there are also entries containing hard-to-find information on contemporary novels, including works by multicultural, international, and women novelists.

The information covered in each entry includes an introduction to the novel and the novel's author; a plot summary, to help readers unravel and understand the events in a novel; descriptions of important characters, including explanation of a given character's role in the novel as well as discussion about that character's relationship to other characters in the novel; analysis of important themes in the novel; and an explanation of important literary techniques and movements as they are demonstrated in the novel.

In addition to this material, which helps the readers analyze the novel itself, students are also provided with important information on the literary and historical background informing each work. This includes a historical context essay, a box comparing the time or place the novel was written to modern Western culture, a critical overview essay, and excerpts from critical essays on the novel. A unique feature of NCfS is a specially commissioned critical essay on each novel, targeted toward the student reader.

To further aid the student in studying and enjoying each novel, information on media adaptations is provided, as well as reading suggestions for works of fiction and nonfiction on similar themes and topics. Classroom aids include ideas for research papers and lists of critical sources that provide additional material on the novel.

Selection Criteria

The titles for each volume of NCfS were selected by surveying numerous sources on teaching literature and analyzing course curricula for various school districts. Some of the sources surveyed included: literature anthologies; Reading Lists for College-Bound Students: The Books Most Recommended by America's Top Colleges; textbooks on teaching the novel; a College Board survey of novels commonly studied in high schools; a National Council of Teachers of English (NCTE) survey of novels commonly studied in high schools; the NCTE's Teaching Literature in High School: The Novel;and the Young Adult Library Services Association (YALSA) list of best books for young adults of the past twenty-five years. Input was also solicited from our advisory board, as well as educators from various areas. From these discussions, it was determined that each volume should have a mix of □classic□ novels (those works commonly taught in literature classes) and contemporary novels for which information is often hard to find. Because of the interest in expanding the canon of literature, an emphasis was also placed on including works by international, multicultural, and women authors. Our advisory board members□educational professionals□ helped pare down the list for each volume. If a work was not selected for the present volume, it was often noted as a possibility for a future volume. As always, the editor welcomes suggestions for titles to be included in future volumes.

How Each Entry Is Organized

Each entry, or chapter, in NCfS focuses on one novel. Each entry heading lists the full name of the novel, the author's name, and the date of the novel's publication. The following elements are contained in each entry:

- Introduction: a brief overview of the novel which provides information about its first appearance, its literary standing, any controversies surrounding the work, and major conflicts or themes within the work.
- Author Biography: this section includes basic facts about the author's life, and focuses on events and times in the author's life that inspired the novel in question.
- Plot Summary: a factual description of the major events in the novel. Lengthy summaries are broken down with subheads.
- Characters: an alphabetical listing of major characters in the novel. Each character name is followed by a brief to an extensive description of the character's role in the novel, as well as discussion of the character's actions, relationships, and possible motivation. Characters are listed alphabetically by last name. If a character is unnamed□for instance, the narrator in Invisible Man-the character is listed as □The Narrator□ and alphabetized as □Narrator.□ If a character's first name is the only one given, the name will appear alphabetically by that name. □ Variant names are also included for each character. Thus, the full name □Jean Louise Finch□ would head the listing for the narrator of To Kill a Mockingbird, but listed in a separate cross-reference would be the nickname □Scout Finch.□
- Themes: a thorough overview of how the major topics, themes, and issues are addressed within the novel. Each theme discussed appears in a separate subhead, and is easily accessed through the boldface entries in the Subject/Theme Index.
- Style: this section addresses important style elements of the novel, such as setting, point of view, and narration; important literary devices used, such as imagery, foreshadowing, symbolism; and, if applicable, genres to which the work might have belonged, such as Gothicism or Romanticism. Literary terms are explained within the entry, but can also be found in the Glossary.
- Historical Context: This section outlines the social, political, and cultural climate in which the author lived and the novel was created. This section may include descriptions of related historical events, pertinent aspects of daily life in the culture, and the artistic and literary sensibilities of the time in which the work was written. If the novel is a historical work, information regarding the time in which the novel is set is also included. Each section is broken down with helpful subheads.
- Critical Overview: this section provides background on the critical reputation of the novel, including bannings or any other public controversies surrounding the work. For older works, this section includes a history of how the novel was first received and how perceptions of it may have changed over the years; for more recent novels, direct quotes from early reviews may also be included.
- Criticism: an essay commissioned by NCfS which specifically deals with the novel and is written specifically for the student audience, as well as excerpts from previously published criticism on the work (if available).

- Sources: an alphabetical list of critical material quoted in the entry, with full bibliographical information.
- Further Reading: an alphabetical list of other critical sources which may prove useful for the student. Includes full bibliographical information and a brief annotation.

In addition, each entry contains the following highlighted sections, set apart from the main text as sidebars:

- Media Adaptations: a list of important film and television adaptations of the novel, including source information. The list also includes stage adaptations, audio recordings, musical adaptations, etc.
- Topics for Further Study: a list of potential study questions or research topics dealing with the novel. This section includes questions related to other disciplines the student may be studying, such as American history, world history, science, math, government, business, geography, economics, psychology, etc.
- Compare and Contrast Box: an □at-a-glance□ comparison of the cultural and historical differences between the author's time and culture and late twentieth century/early twenty-first century Western culture. This box includes pertinent parallels between the major scientific, political, and cultural movements of the time or place the novel was written, the time or place the novel was set (if a historical work), and modern Western culture. Works written after 1990 may not have this box.
- What Do I Read Next?: a list of works that might complement the featured novel or serve as a contrast to it. This includes works by the same author and others, works of fiction and nonfiction, and works from various genres, cultures, and eras.

Other Features

NCfS includes □The Informed Dialogue: Interacting with Literature,□ a foreword by Anne Devereaux Jordan, Senior Editor for Teaching and Learning Literature (TALL), and a founder of the Children's Literature Association. This essay provides an enlightening look at how readers interact with literature and how Nonfiction Classics for Students can help teachers show students how to enrich their own reading experiences.

A Cumulative Author/Title Index lists the authors and titles covered in each volume of the NCfS series.

A Cumulative Nationality/Ethnicity Index breaks down the authors and titles covered in each volume of the NCfS series by nationality and ethnicity.

A Subject/Theme Index, specific to each volume, provides easy reference for users who may be studying a particular subject or theme rather than a single work. Significant subjects from events to broad themes are included, and the entries pointing to the specific theme discussions in each entry are indicated in boldface.

Each entry has several illustrations, including photos of the author, stills from film adaptations (if available), maps, and/or photos of key historical events.

Citing Nonfiction Classics for Students

When writing papers, students who quote directly from any volume of Nonfiction Classics for Students may use the following general forms. These examples are based on MLA style; teachers may request that students adhere to a different style, so the following examples may be adapted as needed. When citing text from NCfS that is not attributed to a particular author (i.e., the Themes, Style, Historical Context sections, etc.), the following format should be used in the bibliography section:

□Night.□ Nonfiction Classics for Students. Ed. Marie Rose Napierkowski. Vol. 4. Detroit: Gale, 1998. 234-35.

When quoting the specially commissioned essay from NCfS (usually the first piece under the □Criticism□ subhead), the following format should be used:

Miller, Tyrus. Critical Essay on □Winesburg, Ohio.□ Nonfiction Classics for Students. Ed. Marie Rose Napierkowski. Vol. 4. Detroit: Gale, 1998. 335-39.

When quoting a journal or newspaper essay that is reprinted in a volume of NCfS, the following form may be used:

Malak, Amin. □Margaret Atwood's □The Handmaid's Tale and the Dystopian Tradition,□ Canadian Literature No. 112 (Spring, 1987), 9-16; excerpted and reprinted in Nonfiction Classics for Students, Vol. 4, ed. Marie Rose Napierkowski (Detroit: Gale, 1998), pp. 133-36.

When quoting material reprinted from a book that appears in a volume of NCfS, the following form may be used:

Adams, Timothy Dow. □Richard Wright: □Wearing the Mask,□ in Telling Lies in Modern American Autobiography (University of North Carolina Press, 1990), 69-83; excerpted and reprinted in Novels for Students, Vol. 1, ed. Diane Telgen (Detroit: Gale, 1997), pp. 59-61.

We Welcome Your Suggestions

The editor of Nonfiction Classics for Students welcomes your comments and ideas. Readers who wish to suggest novels to appear in future volumes, or who have other suggestions, are cordially invited to contact the editor. You may contact the editor via email at: ForStudentsEditors@gale.com. Or write to the editor at:

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